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Full Operator Structure of the Nonleptonic $|\Delta S| = 1$ Weak Hamiltonian

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ABSTRACT

We give a detailed analysis of the $m\bar{s}\sigma_{\mu\nu}\frac{\lambda^A}{2}G^{\mu\nu A}d$ operators that contribute to $|\Delta S| = 1$, $|\Delta I| = ½$ amplitudes in the $SU(3)_{color} \times SU(2) \times U(1)$ gauge theory. We compute and compare the renormalization group improved coefficients of this and all other operators involved in the Hamiltonian paying particular care to the u - c GIM cancellation and the relevant short-distance scales. We give a brief discussion of matrix elements in the valence quark approximation.

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I. INTRODUCTION

In a recent letter[1] we have reanalyzed the coefficients of the dimension-6 local operators occuring in the $|\Delta S| = 1$ nonleptonic weak Hamiltonian, paying particular regard to the u - c GIM cancellation which affects the so-called "penguin" contributions. We have studied the log structure which occurs generally in multi-loop processes and find that the QCD radiative corrections for "penguins" are modified from those given in the standard analysis.[2] This widens the gap between the experimental values and the theoretical results when one assumes the matrix element estimates of ref. (2), and we are further led to argue for substantial modifications in the values of operator matrix elements. Hence, the $\Delta I = ½$ rule, if it can be said to be a theoretical "prediction" at all, appears to be due to the cumulative effects of the many $|\Delta I| = ½$ operators that are induced by QCD in the $SU(2) \times U(1)$ electroweak interactions, rather than the dominant effect of a single operator.

We must be careful to include the effects of all local operators that can occur in the nonleptonic weak Hamiltonian. Presently we extend to completion the analysis of operator contributions by computing at the two-loop level the coefficients and anomalous dimensions of the d = 5 operators needed to complete the basis. In a purely left-handed weak interaction theory one must go to the two-loop processes of Fig. 1 including the mass insertions of s and d quarks. The only new operators we need consider are of the generic form of a "glue-anomalous magnet moment" and can be written:

$$\mathcal{O} \sim m\bar{s}\sigma_{\mu\nu} \frac{\lambda^A}{2} G^{\mu\nu A} d \qquad (1)$$

where $G^{\mu\nu A}$ is the gluon field strength (we specify the exact mass and chiral structures below). Such operators have been considered previously in the context of vectorlike theories[3-5] and as arising by Higgs boson exchange.[6]

In the purely left-handed theories estimates have been performed by including mass insertions external to 1-loop diagrams.[2] An exact calculation in two-loops is performed presently. This is a nonleading log contribution at the two-loop level, the leading log vanishing by chirality. There are subtleties that arise in connection with the Becchi-Rouet-Stora (BRS) Ward identity which allows the existence of gauge non-invariant counterterms in the diagrams of Fig. (1, 2). These difficulties can be circumvented easily by using the Gordon-decomposition of the $|\Delta S| = 1$ weak current, which includes mass effects of the form in eq. (1), but which allows computing only terms other than the $\sigma_{\mu\nu}$ vertex. This is effectively equivalent to working "on-shell" in which case the BRS-allowed gauge non-invariant counterterms vanish. We are careful, however, to check this result with a detailed "off-shell" calculation in which we construct and isolate the effects of the gauge non-invariant counterterms. This technical analysis is relegated to the appendix. Our resulting coefficients and anomalous dimensions have been explicitly checked for gauge (α) invariance in a covariant gauge.

The resulting coefficient of this new operator, including the QCD radiative corrections summed by the renormalization group, is obtained and compared to the previous calculation of all other $d = 6$ operators of ref. (1). We find that the resulting coefficient is roughly two orders of magnitude smaller than the largest "penguin" coefficient. We give a crude estimate of matrix elements in Section IV. This leads to an extra factor of $\bar{g}(1\text{ GeV})$ in the contribution of the new operator and hence, the net effect is in the range (1/10 to 1/100) that of penguins. Hence, it is unlikely that the $d = 5$ operator contributes significantly to the nonleptonic weak Hamiltonian. Furthermore, higher order QCD effects are probably small in general.

We feel, however, that the analysis of the full operator structure including the operator of eq. (1) is an interesting and illustrative example in the application of QCD to experimentally accessible problems.

II. OPERATORS

The effective nonleptonic weak Hamiltonian has the form:

$$H_{wk} = \sqrt{2} G_F \cos\theta_c \sin\theta_c \sum_j C_j(M_W^2, m^2, \alpha_s, \mu^2) \mathcal{O}^j_{\mu^2} \tag{2}$$

where the coefficient functions C_j in general depend upon the W-boson mass, M_W, and any other (e.g. quark) mass scales, together with the strong coupling constant, $\alpha_s = g^2/4\pi$, and an operator normalization point μ^2. The operators are ordered according to their dimension; the leading contribution coming from $d \leq 6$ operators. $|\Delta S| = 1$ operators of $d = 4$ do not appear after renormalization.

There are many candidate $d \leq 6$ $|\Delta S| = 1$ operators that could occur in eq. (1), but upon use of algebraic relationships and equations of motion one finds a minimal basis of independent $d = 6$ four quark operators and two $d = 5$ quark-quark-gluon operators.[5] These are the following:

$$\mathcal{O}_1^c = \bar{s}\gamma_\mu d_L \bar{c}\gamma^\mu c_L - \bar{s}\gamma_\mu c_L \bar{c}\gamma^\mu d_L$$

$$\mathcal{O}_2^c = \bar{s}\gamma_\mu d_L \bar{c}\gamma^\mu c_L + \bar{s}\gamma_\mu c_L \bar{c}\gamma^\mu d_L$$

$$\mathcal{O}_1^u = \bar{s}\gamma_\mu d_L \bar{u}\gamma^\mu u_L - \bar{s}\gamma_\mu u_L \bar{u}\gamma^\mu d_L$$

$$\mathcal{O}_2^{u} = \bar{s}\gamma_\mu d_L \bar{u}\gamma^\mu u_L + \bar{s}\gamma_\mu u_L \bar{u}\gamma^\mu d_L + 2\bar{s}\gamma_\mu d_L \bar{d}\gamma^\mu d_L + 2\bar{s}\gamma_\mu d_L \bar{s}\gamma^\mu s_L$$

$$\mathcal{O}_5 = \bar{s}\gamma_\mu \lambda^A d_L (\bar{u}\gamma^\mu \lambda^A u_R + \bar{d}\gamma^\mu \lambda^A d_R + \bar{s}\gamma^\mu \lambda^A s_R + \bar{c}\gamma^\mu \lambda^A c_R)$$

$$\mathcal{O}_6 = \bar{s}\gamma_\mu d_L (\bar{u}\gamma^\mu u_R + \bar{d}\gamma^\mu d_R + \bar{s}\gamma^\mu s_R + \bar{c}\gamma^\mu c_R)$$

$$\mathcal{O}_7^{\pm} = m_s \bar{s}_R \sigma_{\mu\nu} \frac{\lambda^A}{2} d_L G^{\mu\nu A} \pm m_d \bar{s}_L \sigma_{\mu\nu} \frac{\lambda^A}{2} d_R G^{\mu\nu A} \tag{3}$$

where we have followed ref. (2) ($\mathcal{O}_3$ and $\mathcal{O}_4$ are omitted, $\Delta I = 3/2$ operators). λ^A are Gell-Man SU(3) matrices and $G^{\mu\nu A}$ is the gluon field strength tensor.

The coefficients and anomalous dimensions of the operators $\{\mathcal{O}_1^c, \ldots \mathcal{O}_4\}$ have been computed and used elsewhere.[1,2] These calculations are not affected by the presence of $\mathcal{O}_7^{\pm}$ because the mixing matrix is triangular. This is a consequence of the fact that $d = 6$ operators can mix down into $d = 5$, but not vice versa; triangularity guarantees that the $d = 6$ eigenvalues do not change when $\mathcal{O}_7^{\pm}$ is included.

Our task is to include the extra effects of $\mathcal{O}_7^{\pm}$ by computing the coefficients, $C_7^{\pm}$, with full QCD radiative corrections, summed by the renormalization group. In the standard $SU(3)_c \times SU(2) \times U(1)$ model, $\mathcal{O}_7^{\pm}$ cannot appear in the Hamiltonian, eq. (1), until two loops as in Fig. (1). We will compute the coefficients of $\mathcal{O}_7^{\pm}$ and the anomalous dimensions for $\{\mathcal{O}_1^c, \ldots, \mathcal{O}_4\} \rightarrow \mathcal{O}_7^{\pm}$ in the approximation of neglecting $1/N$ terms relative to leading N terms in SU(N) of color. $C_7^{\pm}$ will be obtained by including the QCD effects that mix $\{C_1^c, \ldots, C_4\} \rightarrow C_7$.

The diagrams of Fig. (1,2) are actually finite by the GIM cancellation so we must treat mixing from $d = 6$ operators into $d = 5$ with great care. As discussed in ref. (1), by explicit calculation of the logs occuring in two-loop Feynman diagrams, we expect QCD corrections to go as $(g^2 \log m_c{}^2/\mu^2)^p$ for those operators that involve a u - c cancellation. This differs from the treatment of ref. (2) in which terms like $(g^2)^{p+q}(\log m_c{}^2/\mu^2)^p(\log M_W{}^2/\mu^2)^q$ are encountered.

In the following section we obtain the desired renormalization group improvements for $C_7^{\pm}$ by integrating the relevant anomalous dimensions between $m = \mu$ to $m \simeq m_c$. We discuss the simplified evaluation of the new operator anomalous dimensions.

III. COMPUTATIONS

The actual calculation of the coefficients and anomalous dimensions for $\{d = 6\} \rightarrow \mathcal{O}_7^{\pm}$ in the diagrams of Fig. 1 is first simplified by using operator techniques and to compute, instead, the diagrams of Fig. 2. For our particular operator this produces a result valid to $\mathcal{O}(1/M_W{}^4)$.

The calculation is complicated by the effects of BRS allowed gauge non-invariant counterterms and a naive calculation of the $\sigma_{\mu\nu} q^\nu$ vertex in Fig. 2 produces an incorrect result and one which is α-dependent in a covariant gauge. This is due to the fact that a part of this term resides in a gauge non-invariant operator counterterm. These problems are circumvented by working "on-shell" (as far as diagram numerators are concerned) by using the Gordon decomposition of the current to write:

$$i\sigma_{\mu\nu} q^\nu = -(p + p')_\mu + (\text{masses}) \times \gamma_\mu$$

and then extract the coefficient of the $(p + p')_\mu$ term. This is essentially representing the use of quark equations of motion in all diagrams, whence the gauge non-invariant operators vanish.

We find that the $(p + p')_\mu$ term has a coefficient of $\not{q}$ in Fig. (2) which produces an "on-shell" result: $m_s(\bar{s}_R(\sigma)d_L) - m_d(\bar{s}_L(\sigma)d_R)$. This is a quick way of seeing that only $\mathcal{O}_7^{(-)}$ is involved in the weak Hamiltonian at the two-loop level and that $\mathcal{O}_7^{(+)}$ has zero coefficient and mixing.

We have carefully verified that this result is consistent with an "off-shell" analysis and produces an α-independent result in covariant gauge QCD. Here one constructs all allowed gauge non-invariant counterterms consistent with the BRS Ward identity.[8] We then extract the coefficient of the physical operators $\mathcal{O}_7^{\pm}$. There are seen to be terms superficially associated with the structure of $\mathcal{O}_7^{(+)}$, but in fact these terms are part of the gauge non-invariant counterterm $Y_4 + Y_5$ (eq. (A.1)). The gauge non-invariant operators can have no physical effects; their anomalous dimensions are α-dependent in general, but mixing to them is triangular and they have zero matrix elements.[8] The anomalous dimensions for $\mathcal{O}_7^{\pm} \to \mathcal{O}_7^{\pm}$ are obtained in ref. (4,5,6) where similar subtleties also arise.

The resulting anomalous dimension matrix for the operators of eq. (3) (acting upon coefficient functions) is the following:

$$\gamma(g, m^2, \mu^2) = -\frac{g^2}{16\pi^2}\hat{\gamma} \tag{4}$$

where, in the approximation of ignoring quark masses, $\hat{\gamma}$ takes the form:

$$\begin{bmatrix} 4-\frac{2}{9} & \frac{2}{9} & -\frac{2}{9} & \frac{10}{9} & \frac{16}{9} & 0 & 0 \\ \frac{2}{9} & -2-\frac{2}{9} & \frac{2}{9} & -\frac{10}{9} & -\frac{8}{9} & 0 & 0 \\ -\frac{2}{9} & \frac{2}{9} & 4-\frac{2}{9} & \frac{10}{9} & \frac{16}{9} & 0 & 0 \\ \frac{1}{9} & -\frac{1}{9} & \frac{1}{9} & -2-\frac{5}{9} & -\frac{8}{9} & 0 & 0 \\ \frac{1}{6} & -\frac{1}{6} & \frac{1}{6} & -\frac{5}{6} & 6-\frac{1}{3} & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{16}{3} & 0 & 0 \\ \frac{2}{3}f & -\frac{2}{3}f & \frac{2}{3}f & -\frac{10}{3}f & -\frac{4}{3}f & 6f & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} C_1^c \\ C_2^c \\ C_1^u \\ C_2^u \\ C_5 \\ C_6 \\ C_7 \end{bmatrix} \tag{5}$$

Here, f is defined by

$$f = \frac{g}{16\pi^2}\left(\frac{3}{8}\right) \tag{6}$$

as determined by the evaluation of Fig. (2).

The renormalized coefficient functions $\tilde{C}_i$ are obtained as the following solution to the renormalization group equations:

$$\tilde{C}_i = \sum_j T\left(\exp \int_{\mu^2}^{\mu_i^2} \gamma(\bar{g}, m^2)\frac{dm^2}{m^2}\right)_{ij} C_j^{(0,i)} \tag{7}$$

where $C_j^{(0,i)}$ is an appropriate boundary condition for the ith coefficient and μ_i^2 is an appropriate upper mass-squared limit. For the coefficients $\tilde{C}_1^u$, $\tilde{C}_2^u$, $\tilde{C}_1^c$, $\tilde{C}_2^c$ we follow the usual Lee-Gaillard prescription which agrees with the fact that M_W^2 sets the scale of short distances for these operators and the boundary conditions are just those coefficients obtained in the tree approximation:

$$C_1^{0,u} = -1 \ , \ C_2^{0,u} = \frac{1}{5} \ , \ C_1^{0,c} = 1 \ , \ C_2^{0,c} = -1 \ , \ C_5^0 = C_6^0 = C_7^0 = 0 \ . \tag{8}$$

For the renormalized coefficients $\tilde{C}_5$, $\tilde{C}_6$ and $\tilde{C}_7$ we must be more careful. We must include the mass effects in the anomalous dimension matrix and carefully isolate the short-distance scale. For the sliding mass scale range $\mu^2 < m^2 < m_c^2$ only up quark loops should be included in the anomalous dimension matrix, whereas for $m^2 > m_c^2$ the charm quark effects "turn on." This is the picture of ref. (2) and it can in principle be made rigorous by including mass effects explicitly in γ.

We depart from the analysis of ref. (2) however in the choice of μ_i^2 for i = 5, 6 and 7. The explicit analysis of two-loop (and multi-loop) Feynman diagrams of ref. (1) indicates that it is only m_c^2 and not M_W^2 that sets the scale of short-distance for both penguin and sigma operators. Hence, the renormalization group must sum a series in $g^{2p}(\log m_c^2/\mu^2)^p$ as opposed to the inclusion of $g^{2(p+q)}(\log m_c^2)^p(\log M_W^2)^q$ terms encountered in (2). Hence, we obtain for the coefficients $\tilde{C}_{5,6,7}$:

$$\tilde{C}_i = -T\left(\exp\int_{\mu^2}^{m_c^2} \gamma\frac{dm^2}{m^2}\right)_{ij}\hat{C}_j \quad (i = 5,6,7) \tag{9}$$

where

$$\hat{C}_1^c = \hat{C}_2^c = 0 \; ; \; \hat{C}_1^u = -1 \; ; \; \hat{C}_2^u = \frac{1}{5} \; ; \; \hat{C}_5 = \hat{C}_6 = \hat{C}_7 = 0 \quad . \tag{10}$$

It is convenient to go into a new basis in which the 7×7 γ matrix of eq. (5) is reduced. We define:

$$\begin{aligned} &A = \mathcal{O}_1^u - \mathcal{O}_1^c \qquad B = \mathcal{O}_2^u - 5\,\mathcal{O}_2^c \\ &C = \mathcal{O}_1^u + \mathcal{O}_1^c \qquad D = \mathcal{O}_2^u + 2\,\mathcal{O}_2^c;\; \mathcal{O}_5,\; \mathcal{O}_6,\; \mathcal{O}_7^{(-)} \end{aligned} \tag{10}$$

in which the anomalous dimension matrix $\hat{\gamma}$ is block diagonal:

$$\hat{\gamma} = \begin{bmatrix} \hat{\gamma}_1 & 0 \\ 0 & \hat{\gamma}_2 \end{bmatrix}$$

where,

$$\hat{\gamma}_1 = \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} C_A \\ C_B \end{bmatrix} \tag{11}$$

and

$$\hat{\gamma}_2 = \begin{bmatrix} \frac{32}{9} & \frac{14}{9} & \frac{16}{9} & 0 & 0 \\ \frac{2}{9} & -\frac{25}{9} & -\frac{8}{9} & 0 & 0 \\ \frac{1}{3} & -\frac{7}{6} & \frac{17}{3} & \frac{3}{2} & 0 \\ 0 & 0 & \frac{16}{3} & 0 & 0 \\ \frac{4}{3}f & -\frac{14}{3}f & -\frac{4}{3}f & 6f & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} C_C \\ C_D \\ C_5 \\ C_6 \\ C_7 \end{bmatrix} \tag{12}$$

We calculate C_1^u and C_2^u using the coefficients of eq. (8) and $\mu_i^2 = M_W^2$. This gives $C_A^o = -1$ and $C_B^o = 1/5$ and we need only the terms involving $\hat{\gamma}_1$ in eq. (7) since $C_D^o = C_C^o = C_5^o = C_6^o = C_7^o = 0$. To calculate $\tilde{C}_5$, $\tilde{C}_6$ and $\tilde{C}_7$ we need only the up quark loop contributions with coefficients given by eq. (10). This gives $C_C^o = -1/2$, $C_D^o = 1/7$ and for $\tilde{C}_5$, $\tilde{C}_6$ and $\tilde{C}_7$ we need only use the terms involving $\hat{\gamma}_2$ in eq. (9).

We evaluate eq. (7) directly by computer writing

$$\tilde{C}^i = \sum_j \prod_{n=1}^{N} \left(1 + \frac{\delta M_n^{\ 2}}{M_n^{\ 2}} \gamma(\bar{g}^2(M_n)) \right)_{ij\ (\mathrm{ordered})} C_j^{(0)} \tag{13}$$

with $M_N^{\ 2} = M_i^{\ 2}$ and $M_1^{\ 2} = \mu^2$, and the product is ordered with the largest $M_n^{\ 2}$ (smallest $\bar{g}(M_n)$) to the right. We evaluate the $\tilde{C}_i$ using the above methods for different choices of $m_c^{\ 2}$ and Λ (the strong coupling constant parameter). These are presented in Table I. We have also recalculated the coefficients C_5 and C_6 using the methods of ref. (2) with our parameters $m_c^{\ 2}$, Λ for comparison in Table II. For the usual choice, $\Lambda = 500$ meV, we have $\alpha_s(1\text{ GeV}) \simeq 1$. Varying Λ may be regarded as varying μ with fixed Λ.

We see that $\tilde{C}_7$ is an order of magnitude smaller than $\tilde{C}_6$. In the following section we will estimate the matrix elements of the operator $\mathcal{O}_7^{(-)}$.

IV. HAMILTONIAN AND MATRIX ELEMENTS

The effective nonleptonic weak Hamiltonian is the following:

$$H_{wk} = \sqrt{2} G_F \cos\theta_c \sin\theta_c \ \{ \sum_j \tilde{C}_j \mathcal{O}_j \} \tag{14}$$

where $\tilde{C}_j$ are presented in Table I under various assumptions about Λ and m_c^2 and M_W^2. In order to compare the Hamiltonian with experiment it is necessary to estimate the matrix elements of the contributing operators. This is unfortunately, the "weak link" in the chain of analysis.

In ref. (2) a valence quark approximation has been used to estimate the contributions of $\mathcal{O}_1^u$, $\mathcal{O}_2^u$, $\mathcal{O}_5$ and $\mathcal{O}_6$ to different $\Delta I = ½$ processes. Large results are reported for the matrix elements of $\mathcal{O}_5$ in that the simultaneous use of the vacuum insertion and quark equations of motion lead to contributions like $m_\pi^2/m_s m_u$. The use of current quark masses, $m_s = 150$, $m_u = 5$ mev enhances these amplitudes enormously. Taken together with the coefficients $\tilde{C}_5$ and $\tilde{C}_6$ estimated by the methods of ref. (2), which we have reproduced for comparison with our assumptions about Λ and m_c^2 in Table II, agreement with experiment is obtained to within a factor of 2. Unfortunately, this gap widens if we use our revised estimates for $\tilde{C}_5$ and $\tilde{C}_6$ in Table I. In ref. (1) we argue that $\mathcal{O}_5$ does not have large matrix elements since the constituent quark masses are relevant in its estimates and this is roughly equivalent to the bag model results.

How large are the effects of $\mathcal{O}_7^{(-)}$? Because $\mathcal{O}_7^{(-)}$ includes the gluon field explicitly, there will be a gluonic contribution in any process, e.g. $K \to 2\pi$ as indicated in Fig. (3a). In addition, however, there is the valence quark contribution of Fig. (3b) which we now briefly discuss. From Fig. (3b) we have the typical amplitude:

$$\mathscr{A} = \tilde{C}_7^{(-)} \langle K | \frac{\bar{g}}{q^2}(m_s \bar{s}_R \sigma_{\mu\nu} q^\nu \frac{\lambda^A}{2} d_L - m_d \bar{s}_L \sigma_{\mu\nu} q^\nu \frac{\lambda^A}{2} d_R) j^{\mu A} | \pi \rangle$$

$$= \tilde{C}_7^{(-)} \langle K | \left(\frac{\bar{g}}{q^2} \right) \Big\{ \left((m_s^2 - m_d^2) \bar{s} \gamma_\mu \frac{\lambda^A}{2} d_L + 2 m_s m_d \bar{s} \gamma_\mu \frac{\lambda^A}{2} d_R \right) j^{\mu A} \quad (15)$$

$$- (m_s \bar{s} \frac{\lambda^A}{2} d_L + m_d \bar{s} \frac{\lambda^A}{2} d_R)(p + p')_\mu j^{A\mu} \Big\} | \pi \rangle$$

where:

$$j_\mu^{\ A} = \bar{u} \gamma_\mu \frac{\lambda^A}{2} u + \bar{d} \gamma_\mu \frac{\lambda^A}{2} d + \ldots \quad (16)$$

If we consider just the term in (16) containing m_s^2 we have

$$\mathscr{A} = \tilde{C}_7^{(-)} \langle \frac{m_s^2}{q^2} \bar{g} \rangle \langle \bar{s} \gamma_\mu \frac{\lambda^A}{2} d_L j^{\mu A} \rangle + \ldots \quad (17)$$

Note that the m_s^2/q^2 is very sensitive to the long-wavelength (e.g. bag radius) components of hadrons. Also, $\bar{g}$ is evaluated at q^2. Using a Fierz rearrangement and assuming all mass scales are comparable we obtain:

$$\langle \tilde{C}_7 \mathscr{O}_7^{(-)} \rangle \sim \tilde{C}_7 \langle \frac{m_s^2}{q^2} \bar{g}(q^2) \rangle \langle u \rangle \sim \bar{g}(1) \tilde{C}_7 \langle u \rangle \quad (18)$$

where $\langle u \rangle \approx 1$. The gluonic contribution is harder to estimate. In ref. (3) it was argued that the ratio $\Gamma(s \to d + \text{gluon})/\Gamma(s \to u\bar{u}d)$ calculated in the free quark model might give some estimate of this term. This would imply C_7 should be multiplied by a factor of the order of 40 when comparing with $\langle C_1 \mathscr{O}_1 \rangle$ or $\langle C_2 \mathscr{O}_2 \rangle$, or a factor of about 5 when comparing with $\mathscr{O}_5$ or $\mathscr{O}_6$. This is not far off the valence quark estimate of about a factor of 3 relative to $\mathscr{O}_5$ and $\mathscr{O}_6$. From

Table I we see that with operator matrix elements of this order $\mathcal{O}_7$ will not contribute significantly in $|\Delta S| = 1$, $|\Delta I| = ½$ processes.

In conclusion, we have completed the calculation of the effective $|\Delta S| = 1$ nonleptonic weak Hamiltonian. We find that the coefficient functions of the operators $\mathcal{O}_5$, $\mathcal{O}_6$ and $\mathcal{O}_7^{(-)}$ are determined by a short distance scale of $1/m_c$, and the coefficient of $\mathcal{O}_7^{(+)}$ vanishes to two-loop order. Comparison with experiment requires estimates of the operator matrix elements. The valence quark plus vacuum insertion technique provides estimates of the contributions from the operators $\mathcal{O}_1^u$ and $\mathcal{O}_2^u$ yielding approximately 1/5 of the observed $|\Delta I| = 1/2$ amplitude. The penguin operators $\mathcal{O}_5$ and $\mathcal{O}_6$, using the same technique, will give only 1/10 to 1/20 of the $|\Delta I| = ½$ amplitude. The sigma term, $\mathcal{O}_7^{(-)}$, is finally even smaller yielding optimistically $\sim 1/10$ of the penguin contribution $\mathcal{O}_5$ and therefore only $\sim 1/100$ of the total decay amplitude. Hence, the effects of $\mathcal{O}_7^{(-)}$ are irrelevant for nonleptonic weak and related processes.

Of course, the estimates of operator matrix elements are crude and it is desired to develop better techniques. Recently calculations of these matrix elements have been reported using current algebra and the bag model.[9] These estimates are broadly in agreement with those found by vacuum insertion techniques that employ constituent quark masses. Again, caution is warranted for the results are very sensitive to the particular soft pion continuation employed.

Assuming the operator matrix elements are reasonable, what could explain the discrepancy between theory and experiment in $|\Delta I| = ½$, $|\Delta S| = 1$ processes? One obvious gap in the theoretical analysis is the non-short distance piece. For the operators $\mathcal{O}_1^u$ and $\mathcal{O}_2^u$ the momentum scale important in generating the coefficients C_1^u and C_2^u is $0 < K^2 < M_W^2$. Due to the large logs (M_W^2) which appear in integrating over this range it may be a good approximation to include only the short distance contribution $\mu^2 < K^2 < M_W^2$ in estimating these contributions.

However, as discussed in ref. (1) and section (3) the momentum scale relevant for the coefficients C_5, C_6 and C_7 is only $0 < K^2 < m_c^2$. Our estimates include the range $\mu^2 < K^2 < m_c^2$ where perturbative calculations of the strong corrections hopefully make sense. Since this region only produces logs of m_c^2/μ^2 it is less plausible we can justifiably ignore the integration region $0 < K^2 < \mu^2$; this may give further large contributions to C_5, C_6 and C_7.

Finally, there remains the possibility of substantial contributions from operators of dimension > 6. As discussed in ref. (1) these operators, though not enhanced by large logs, can arise at $\mathcal{O}(1)$ and may, by their sheer abundance, be important. As in ref. (1), we hope that this will not be the case due to their having small matrix elements.

APPENDIX

In this appendix we carry out a detailed "off-shell" analysis, i.e., one in which we do not make use of the quark equations of motion, as a consistency check upon our much quicker "on-shell" calculation reported in the main body of the paper. We work here in Feynman gauge (though we have checked the "on-shell" calculation in Landau gauge and find it to be α-independent). This work illustrates the subtleties of operator renormalization incurred by the complexity of the allowed nongauge-invariant counterterms.

The most general set of Becchi-Rouet-Stora allowed counterterms are the following[8]:

$$
\begin{aligned}
Y_1^{\pm} &= \tfrac{1}{2}\bar{s}((i\not{D} - m_s)\partial^2 \pm \partial^2(i\not{D} - m_d))d \\
Y_2^{\pm} &= \tfrac{ig}{2}\bar{s}((i\not{D} - m_s)\partial_\mu A^\mu \pm A_\mu \partial^\mu(i\not{D} - m_d))d \\
Y_3^{\pm} &= \tfrac{ig}{2}\bar{s}((i\not{D} - m_s)A^\mu \partial_\mu \pm \partial^\mu A_\mu(i\not{D} - m_d))d \\
Y_4^{\pm} &= \tfrac{ig}{2}\bar{s}((i\not{D} - m_s)\not{\partial}\not{A} \pm \not{A}\not{\partial}(i\not{D} - m_d))d \\
Y_5^{\pm} &= \tfrac{ig}{2}\bar{s}((i\not{D} - m_s)\not{A}\not{\partial} \pm \not{\partial}\not{A}(i\not{D} - m_d))d \quad .
\end{aligned}
\qquad \text{(A.1(a,e))}
$$

There are also operators involving two explicit gauge fields, which will have 4-body vertices, but we do not need to know these since we only require the independent 2(quark-quark) and 3 (quark-quark-gluon) vertices. The above operators, $Y_i^{(-)}$, will not be involved in the ordinary weak Hamiltonians as they have the wrong CP. Henceforth we will refer only to $Y_i \equiv Y_i^{(+)}$. Also, the explicit chirality of the s and d fields is irrelevant to the following discussion and may be restored at the end. Note that the ops of eq. (A.1) are just the most general set of null operators ( ).

We also require the "Penguin" operator

$$\mathcal{O}_P = \bar{s}\gamma_\mu \frac{\lambda^A}{2} d(D_\nu G^{\mu\nu})^A \tag{A.2}$$

which is an allowed counterterm in 3-body diagrams, even though it is related to four-quark operators by the gluon equation of motion. In the present calculation we will obtain no new "Penguin" contribution through $\mathcal{O}_P$ in two loops, but rather the component of a null operator involving $\mathcal{O}_P - (\bar{s}\gamma_\mu \frac{\lambda^A}{2} d)(gj^{\mu A})$, where $j^{\mu A}$ is the quark color current, is obtained.

These operators are determined by the diagrams of Fig. 2 (in the absence of quark mass insertions) to order $1/M_W^4$ and to order $1/N^2$ in the $1/N$ expansion of SU(N) of color. In Table III we give the projection of $\mathcal{O}_P$ and Y_i onto the vertices of Fig. 4 (giving coefficients of additional vertices, such as $q_\mu \not{p}$, $p_\mu \not{q}$ etc. is not useful, as the BRS invariance relates these to the vertices shown in the figure). We also give in Table III the results in Feynman gauge for the evaluation of Fig. 2 with a momentum routing as indicated in Fig. 2a. The coefficients of null operators change with different routings whereas the coefficients of physical operators obviously must not (we have checked this by rerouting q to the left in Fig. 2). In quoting the results, as in Table III an overall factor of $g^2/(16\pi^2)\cdot g/(16\pi^2)\cdot(\log \Lambda^2/m_u^2 - \log \Lambda^2/m_c^2)$ is understood. (Note that these are the next to leading logs for two loops; the leading logs only renormalize the original penguin diagrams and are already included in the matrix, eq. (5).)

From the results and coefficients in Table III, the coefficients of the operators, $\mathcal{O}_P$, Y_i, as counterterms are determined:

$$C_{Y_1} = 0 \ ; \ C_{Y_2} = \frac{N}{2} \ ; \ C_{Y_3} = -\frac{3}{2}N \ ; \ C_{Y_4} = \frac{N}{2} \ ; \ C_{Y_5} = \frac{N}{2} \ ; \ C_{\mathcal{O}_1} = \frac{N}{4} \ . \tag{A.3}$$

To obtain the coefficients of $\mathcal{O}_7^{\pm}$, we must include the mass dependent (d = 5) vertices of Fig. 5. These are not a complete set, but are the only required vertices to obtain $C_{\mathcal{O}_{1^+}}$, $C_{\mathcal{O}_{1^-}}$. (We have analyzed extra d = 5 vertices for consistency checks).

The coefficients of these vertices arising from the above operators, and the results of computing the mass insertions in Fig. 2 are given in Table IV. Using the results for C_{Y_4} and C_{Y_5} in (A.4), we see that $C_{\mathcal{O}_{1^\pm}}$ are determined as follows:

$$4C_{\mathcal{O}_{1^+}} = -\frac{N}{2} + \frac{1}{2}(C_{Y_4} + C_{Y_5}) = 0$$

$$4C_{\mathcal{O}_{1^-}} = -\frac{N}{2} + \frac{1}{2}(C_{Y_4} - C_{Y_5}) = -\frac{N}{2} \quad . \tag{A.4}$$

Restoring the chirality and overall factors, we obtain the term:

$$\left(\frac{3}{8}\right)\frac{g^2}{16\pi^2}\cdot\frac{g}{16\pi^2}\cdot\left(m_s\bar{s}_R\sigma_{\mu\nu}\frac{\lambda^A}{2}G^{\mu\nu A}d_L - m_d\bar{s}_L\sigma_{\mu\nu}\frac{\lambda^A}{2}G^{\mu\nu A}d_R\right)\cdot \log\mathscr{A}$$

which occurs in the nonleptonic weak Hamiltonian. Here $\log\mathscr{A}$ has the form $\approx \log\left(m_c^2/(\langle p^2\rangle + m_u^2)\right)$ where $\langle p^2\rangle$ is a typical quark momentum in a hadron.

Given the mixing of an operator to the penguin operator $\mathcal{O}_5$ to be f, the anomalous dimension of the same operator mixing to $\mathcal{O}_7^{(-)}$ is determined to be:

$$\left(\frac{3}{8}\right)\frac{g^3}{(16\pi^2)^2}\cdot 4f \cdot \mathcal{O}_7^{(-)}$$

where we must, as usual, treat the quark mass effects carefully.

Table I

$\tilde{C}_1^{(u)}$	$\tilde{C}_2^{(u)}$	$\tilde{C}_5$	$\tilde{C}_6$	$\tilde{C}_7$	$g(1)\tilde{C}_7$	Λ GeV	m_c GeV
-1.853	.1468	-.0171	-.0019	-.00032	-.00082	.25	2
-2.363	.1299	-.0324	-.0061	-.00073	-.0026	.5	2
-3.2970	.1096	-.0710	-.0219	-.00182	-.0107	.75	2
-1.853	.1468	-.0407	-.0089	-.00060	-.0015	.25	6
-2.363	.1299	-.0751	-.0238	-.00121	-.0043	.5	6
-3.2970	.1096	-.1587	-.0697	-.00264	-.0172	.75	6

For $\tilde{C}_1{}^u$ and $\tilde{C}_2{}^u$ we choose $M_W = 70$ GeV. For all estimates we choose $\mu = 1$ GeV. $\tilde{C}_1{}^c \approx (-\tilde{C}_1{}^u)$ and $\tilde{C}_2{}^c \approx (-5\tilde{C}_2{}^u)$. (Note that we choose different parameters than those used in ref. (1), Table 1.)

Table II

$\tilde{C}_1^{(u)}$	$\tilde{C}_2^{(u)}$	$\tilde{C}_5$	$\tilde{C}_6$	Λ GeV	m_c GeV
-1.923	.1307	-.0234	-.0028	.25	2
-2.492	.1028	-.0498	-.0098	.5	2
-3.589	.0618	-.1278	-.0405	.75	2
-1.898	.0121	-.0528	-.0123	.25	6
-2.454	.0919	-.1055	-.0348	.5	6
-3.530	.0546	-.2488	-.1114	.75	6

Recomputed coefficients for $\mathcal{O}_1 - \mathcal{O}_4$ using the method of ref. (2) for comparison. We assume $M_W = 70$ GeV, and $\mu = 1$ GeV.

Table III

	$\mathcal{O}_P$	Y_1	Y_2	Y_3	Y_4	Y_5	Results
$\not{p}p^2$	0	1	0	0	0	0	0
$g\frac{\lambda^A}{2}q^2\gamma_\mu$	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$\frac{N}{2}$
$g\frac{\lambda^A}{2}q_\mu\not{q}$	-1	0	$\frac{1}{2}$	0	0	0	0
$g\frac{\lambda^A}{2}p^2\gamma_\mu$	0	1	0	0	1	-1	0
$g\frac{\lambda^A}{2}p_\mu\not{p}$	0	0	1	1	0	2	0
$g\frac{\lambda^A}{2}i\varepsilon_{\mu\nu\rho\sigma}q^\nu p^\rho\gamma^\sigma$	0	0	0	0	0	-1	$-\frac{N}{2}$

Coefficients in the operators of the indicated d = 6, 2 and 3-body vertices; $\mathcal{O}_7^\pm$ have zero coefficients. This is the maximal set of independent d = 6 vertices.

Table IV

	Y_2	Y_3	Y_4	Y_5	$\mathcal{O}_7^{(+)}$	$\mathcal{O}_7^{(-)}$	Results
$\frac{1}{4}(m_s + m_d)[\not{q}, \gamma_\mu]$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	4	0	$-\frac{N}{2}$
$\frac{1}{4}(m_s - m_d)[\not{q}, \gamma_\mu]$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	4	$\frac{N}{2}$

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FIGURE CAPTIONS

Fig. 1: Some two loop diagrams leading to $\mathcal{O}_7^{\pm}$.

Fig. 2: The complete set of diagrams with leading (N) nonvanishing contributions in $1/N$ expansion.

Fig. 3: Matrix element estimates of $\mathcal{O}_7^{\pm}$.

Fig. 4: The independent $d = 6$ 2- and 3-body vertices.

Fig. 5: The required $d = 5$ 3-body vertices.

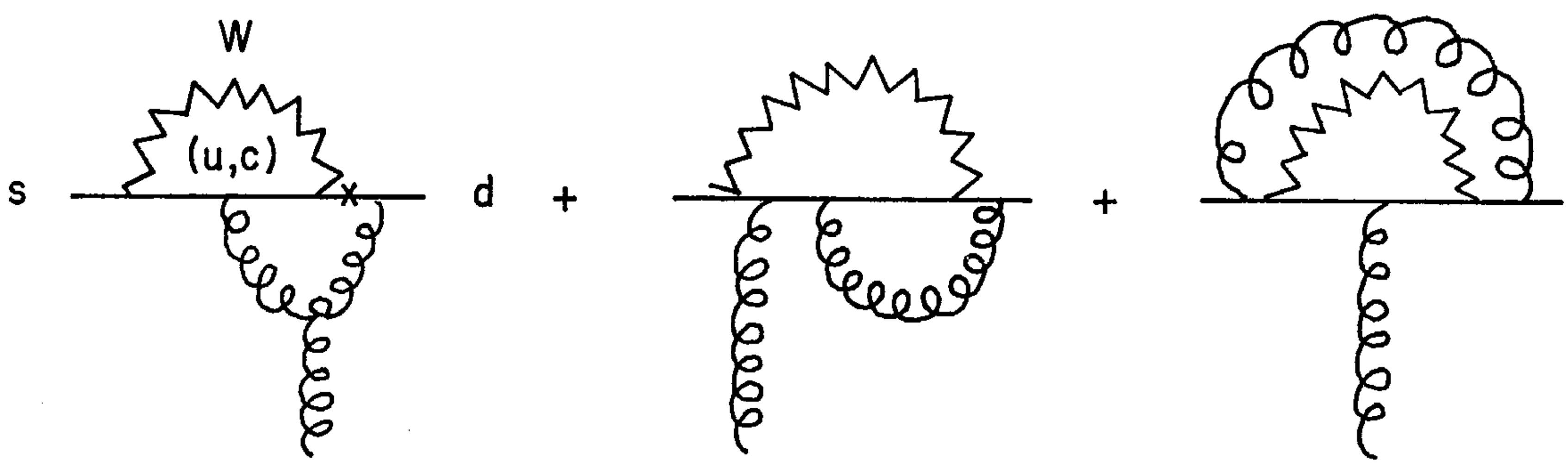


Fig. I

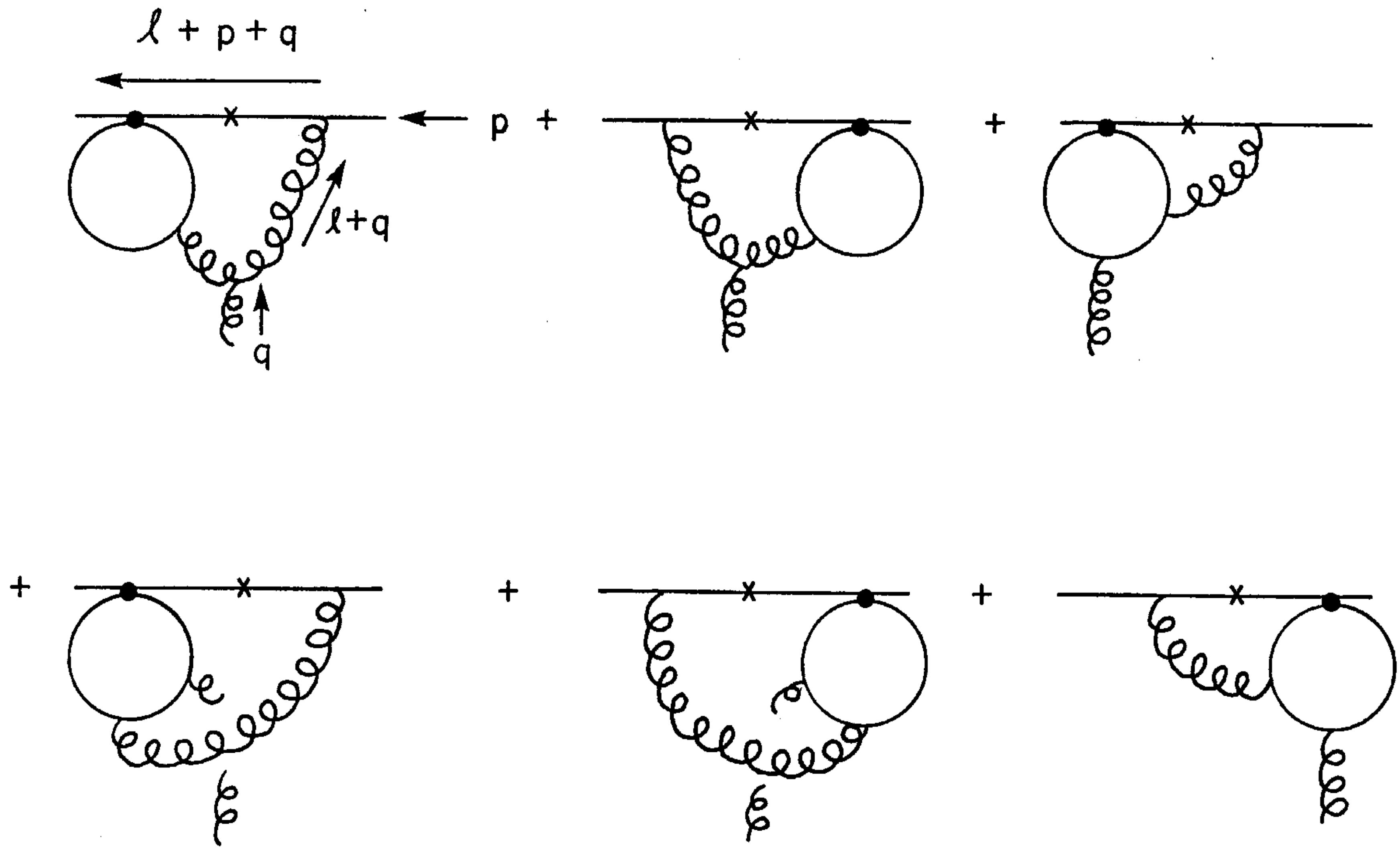


Fig. 2

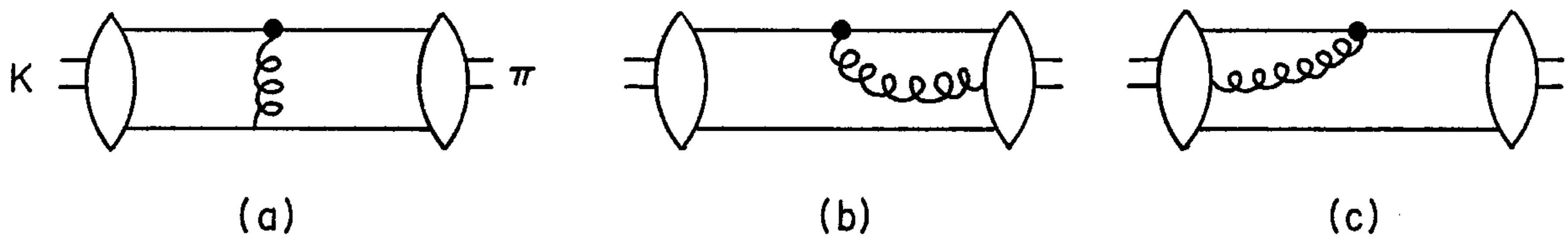
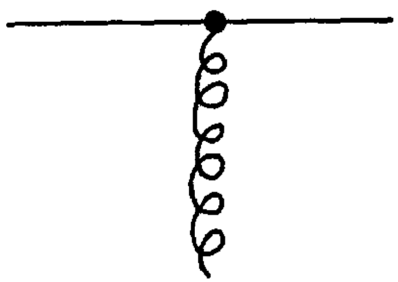


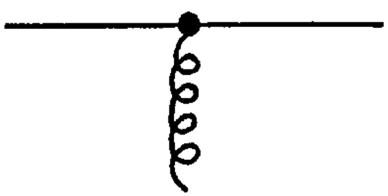
Fig. 3

$\not{p}\, p^2$



$g \frac{\lambda^A}{2} \left(q^2 \gamma_\mu,\ q_\mu \not{q},\ p^2 \gamma_\mu,\ p_\mu \not{p},\ \mathcal{E} \equiv i\, \varepsilon_{\mu\nu\rho\sigma}\, q^\nu p^\rho \gamma^\sigma \gamma_5 \right)$

Fig. 4



$$\left(\frac{1}{4}\,(m_s + m_d)\,[\not{q}, \gamma_\mu],\ \frac{1}{4}\,(m_s - m_d)\,[\not{q}, \gamma_\mu]\right) g\,\frac{\lambda^A}{2}$$

Fig.5